NANYANG TECHNOLOGICAL UNIVERSITY
SYLLABUS FOR ENTRANCE EXAMINATION
FOR INTERNATIONAL STUDENTS

AO-LEVEL MATHEMATICS

STRUCTURE OF EXAMINATION PAPER

1. There will be one 2-hour paper consisting of 4 questions.

2. Each question carries 25 marks.

3. Candidates will be required to answer all 4 questions.

Electronic Calculators

1. The use of common electronic scientific calculators is **allowed.**

The detailed syllabus is on the next page.

Nanyang Technological University
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CONTENT OUTLINE

Knowledge of the content of the Ordinary Level Syllabus (or an equivalent syllabus) is assumed. The use of a scientific calculator, where appropriate, is expected.

	Topic / Sub-topics	Content
PURE MATHEMATICS		
1	**Functions and graphs**	
1.1	Exponential and logarithmic functions and Graphing techniques	Include: • concept of function • use of notation such as $f(x) = x^2 + 5$ • functions e^x and $\ln x$ and their graphs • laws of logarithms • equivalence of $y = e^x$ and $x = \ln y$ • use of a graphic calculator to graph a given function • characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes Exclude: • concepts of domain and range • the use of notation $f: x \mapsto x^2 + 5$
1.2	Equations and inequalities	Include: • solving simultaneous equations, one linear and one quadratic, by substitution • conditions for a quadratic equation to have real or equal roots • solving quadratic inequalities • conditions for $ax^2 + bx + c$ to be always positive (or always negative) • solving inequalities by graphical methods • formulating an equation from a problem situation • finding the numerical solution of an equation using a graphic calculator

	Topic / Sub-topics	Content
2	**Calculus**	
2.1	Differentiation	Include: • derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point • use of standard notations $f'(x)$ and $\frac{dy}{dx}$ • derivatives of x^n for any rational n, e^x, ln x, together with constant multiples, sums and differences • use of chain rule • graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$ • stationary points (local maximum and minimum points and points of inflexion) • finding the numerical value of a derivative at a given point using a graphic calculator • finding equations of tangents and normals to curves • solving practical problems involving differentiation Exclude: • differentiation from first principles • derivatives of products and quotients of functions • use of $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ • differentiation of functions defined implicitly or parametrically • finding non-stationary points of inflexion • problems involving small increments and approximation • relating the graph of $y = f'(x)$ to the graph of $y = f(x)$
2.2	Integration	Include: • integration as the reverse of differentiation • integration of x^n, for any rational n, and e^x, together with constant multiples, sums and differences • integration of $(ax+b)^n$, for any rational n, and $e^{(ax+b)}$ • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • finding the numerical value of a definite integral using a graphic calculator Exclude: • definite integral as a limit of sum • approximation of area under a curve using the trapezium rule • area below the x-axis

	Topic / Sub-topics	Content
STATISTICS		
3	**Probability**	
3.1	Probability	Include: • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities • calculation of conditional probabilities in simple cases • use of $P(A') = 1 - P(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
4	**Binomial and normal distributions**	
4.1	Binomial distribution	Include: • knowledge of the binomial expansion of $(a+b)^n$ for positive integer n • use of the notations $n!$ and $\binom{n}{r}$ • concept of binomial distribution $\mathrm{B}(n,p)$ and use of $\mathrm{B}(n,p)$ as a probability model • use of mean and variance of a binomial distribution (without proof) • solving problems involving binomial variables Exclude calculation of mean and variance for other probability distributions
4.2	Normal distribution	Include: • concept of a normal distribution and its mean and variance; use of $\mathrm{N}(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $\mathrm{P}(X < x_1)$ given the values of x_1, μ, σ • use of the symmetry of the normal distribution • finding a relationship between x_1, μ, σ given the value of $\mathrm{P}(X < x_1)$ • solving problems involving normal variables • solving problems involving the use of $\mathrm{E}(aX + b)$ and $\mathrm{Var}(aX + b)$ • solving problems involving the use of $\mathrm{E}(aX + bY)$ and $\mathrm{Var}(aX + bY)$, where X and Y are independent • normal approximation to binomial Exclude: • finding probability density functions and distribution functions • calculation of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ from other probability density functions

	Topic / Sub-topics	Content
5	**Sampling and hypothesis testing**	
5.1	Sampling	Include: • concepts of population and sample • random, stratified, systematic and quota samples • advantages and disadvantages of the various sampling methods • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large • calculation of unbiased estimates of the population mean and variance from a sample • solving problems involving the sampling distribution
5.2	Hypothesis testing	Include: • concepts of null and alternative hypotheses, test statistic, level of significance and *p*-value • tests for a population mean based on: - a sample from a normal population of known variance - a large sample from any population • 1-tail and 2-tail tests Exclude testing the difference between two population means
6	**Correlation and Regression**	
6.1	Correlation coefficient and linear regression	Include: • concepts of scatter diagram, correlation coefficient and linear regression • calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line • concepts of interpolation and extrapolation Exclude: • derivation of formulae • hypothesis tests • use of a square, reciprocal or logarithmic transformation to achieve linearity

MATHEMATICAL NOTATION

The list which follows summarises the notation used in NTU Entrance Examinations in Mathematics. Although primarily directed towards A-Level, the list also applies, where relevant, at AO-Level.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, {0, ±1, ±2, ±3, ...}
$\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, ...}
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\not>$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\not<$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for n, $r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\frac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

$\sin$, $\cos$, $\tan$, cosec, $\sec$, $\cot$	} the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	} the inverse circular functions

7. *Complex Numbers*

i	square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\ z$	the real part of z, $\mathrm{Re}\ (x + \mathrm{i}y) = x$
$\mathrm{Im}\ z$	the imaginary part of z, $\mathrm{Im}\ (x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta, -\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables

x, y, r, etc.	value of the random variables X, Y, R, etc.
x_1, x_2, ...	observations
f_1, f_2, ...	frequencies with which the observations, x_1, x_2 ...occur
p(x)	the value of the probability function P($X = x$) of the discrete random variable X
p_1, p_2 ...	probabilities of the values x_1, x_2, ...of the discrete random variable X
f(x), g(x)...	the value of the probability density function of the continuous random variable X
F(x), G(x)...	the value of the (cumulative) distribution function P($X \leqslant x$) of the random variable X
E(X)	expectation of the random variable X
E[g(X)]	expectation of g(X)
Var(X)	variance of the random variable X
B(n, p)	binominal distribution, parameters n and p
Po(μ)	Poisson distribution, mean μ
N(μ, σ^2)	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 \quad \frac{1}{n \quad 1}\sum(x \quad \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution N (0, 1)
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample